

# The Energy of Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics

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**Abstract** According to the Einstein, Weinberg, and Møller energy-momentum complexes, we evaluate the energy distribution of the singularity-free solution of the Einstein field equations coupled to a suitable nonlinear electrodynamics suggested by Ayón-Beato and García. The results show that the energy associated with the definitions of Einstein and Weinberg are the same, but Møller not. Using the power series expansion, we find out that the first two terms in the expression are the same as the energy distributions of the Reissner-Nordström solution, and the third term could be used to survey the factualness between numerous solutions of the Einstein field equations coupled to a nonlinear electrodynamics.

**Keywords** Energy · Black hole · Energy-momentum complex · Nonlinear electrodynamics

## 1 Introduction

In general relativity most of the solutions of the Einstein equations exhibit the same important property, which is the existence of singularities [1]. The space-time at the center of a black hole presents an infinite curvature and under the action of infinite gravity the matter is crushed to infinite density. The problem that arises is the impossibility of the laws of physics to hold at the singularity. Important theories, like the Brans-Dicke theory and the Einstein-Cartan theory, couldn't yield a satisfactory solution for avoiding the existence of

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the singularity in their solutions. To avoid the problem of singularity, some regular models have been proposed [2–7]. These solutions are known as “Bardeen black holes” [8], as Bardeen elaborated for the first time an interesting regular black hole model. These models are not exact solutions to Einstein equations, because no known physical source is associated with any of them and the search of the best candidate which can produce singularity-free solutions has continued. A way to solve this problem is to find more general gravity theories avoiding the existence of singularities. String theory [9] produces singularity-free solutions, even at the classical level, due to its intrinsic non-locality. Other examples are given by domain wall solutions with horizons but without singularities in  $\mathcal{N} = 1$  supergravity (cf. [10], and references therein) and exact conformal field theory [11]. Ayón-Beato and García [12, 13] show that in the framework of the standard general relativity it is possible to generate singularity-free solutions of the Einstein field equations coupled to a suitable nonlinear electrodynamics, which in the weak field approximation becomes the usual linear Maxwell theory. The solutions are given by the line element

$$ds^2 = A(r)dt^2 - A(r)^{-1}dr^2 - r^2d\Omega. \tag{1}$$

In this article, we evaluate the energy distributions of above regular black hole solution by using the Einstein, Weinberg, and Møller energy-momentum complexes. Through the paper we use geometrized units ( $G = 1, c = 1$ ), and follow the convention that Latin indices run from 1 to 3 and Greek indices run from 0 to 3.

### 2 Three Pseudotensorial Prescriptions for Two Regular Black Hole Solutions

Let us consider about the first regular black hole solution is presented by Ayón-Beato and García [12] in 1998 with

$$A(r) = 1 - \frac{2mr^2}{(r^2 + q^2)^{3/2}} + \frac{q^2r^2}{(r^2 + q^2)^2}. \tag{2}$$

This solution asymptotically behaves as the Reissner-Nordström solution and the parameters  $m$  and  $q$  represents the mass and the electric charge. At the outset, the energy component of the Einstein energy-momentum complex [14–16] is given by

$$E_{\text{Einstein}} = \frac{1}{16\pi} \int \frac{\partial H_0^{0l}}{\partial x^l} d^3x, \tag{3}$$

where

$$H_0^{0l} = \frac{g_{00}}{\sqrt{-g}} \frac{\partial}{\partial x^m} [(-g)g^{00}g^{lm}]. \tag{4}$$

The energy component of the Einstein energy-momentum complex is most conveniently calculated in the quasi-Cartesian coordinates  $(t, x, y, z)$ . In these coordinates, the line element (1) reads

$$ds^2 = Adt^2 - (dx^2 + dy^2 + dz^2) - \frac{A^{-1} - 1}{r^2}(xdx + ydy + zdz)^2. \tag{5}$$

Then, the required nonvanishing components of the Einstein energy-momentum complex  $H_0^{0l}$  are easily shown in spherical coordinates to be

$$H_0^{0r} = \frac{2\kappa}{r}\hat{r} - \frac{1}{A}\hat{r}(\hat{r} \cdot \nabla A) + \frac{1}{A}\nabla A, \tag{6}$$

where  $\kappa = 1 - A$ . Applying the Gauss theorem we obtain

$$E_{\text{Einstein}} = \frac{1}{16\pi} \oint H_0^{0r} \cdot \hat{r} r^2 d\Omega, \tag{7}$$

and the integral being taken over a sphere of radius  $r$ , with the outward normal  $\hat{r}$  and the differential solid angle  $d\Omega$ . The Einstein energy complex within radius  $r$  reads

$$E_{\text{Einstein}} = \frac{r}{2}(1 - A) = m \left( 1 + \frac{q^2}{r^2} \right)^{-3/2} - \frac{q^2}{2r} \left( 1 + \frac{q^2}{r^2} \right)^{-2} \equiv E_1(r). \tag{8}$$

Next, the Weinberg energy-momentum complex [17–19] is considered as

$$\tau^{\nu\lambda} = \frac{\partial}{\partial x^\rho} Q^{\rho\nu\lambda}, \tag{9}$$

with superpotential

$$Q^{\rho\nu\lambda} = \frac{\partial h_\mu^\mu}{\partial x_\rho} \eta^{\nu\lambda} - \frac{\partial h_\mu^\mu}{\partial x_\nu} \eta^{\rho\lambda} + \frac{\partial h^{\mu\nu}}{\partial x^\mu} \eta^{\rho\lambda} - \frac{\partial h^{\mu\rho}}{\partial x^\mu} \eta^{\nu\lambda} - \frac{\partial h^{\nu\lambda}}{\partial x_\rho} + \frac{\partial h^{\rho\lambda}}{\partial x_\nu}, \tag{10}$$

where  $\eta_{\mu\nu}$  is the Minkowski metric and  $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ . We adopt the convenient convention that the indices on  $h_{\mu\nu}$  and  $\partial/\partial x^\lambda$  are raised and lowered with  $\eta$ . The energy-momentum of the Weinberg energy-momentum complex is most conveniently calculated in the quasi-Cartesian coordinates  $(t, x, y, z)$ , and is given by

$$P^\lambda = \frac{1}{16\pi} \int \frac{\partial Q^{i0\lambda}}{\partial x^i} d^3x. \tag{11}$$

The required nonvanishing components  $Q^{i00}$  of the Weinberg energy complex are easily shown in spherical coordinates to be

$$Q^{i00} = \frac{\eta}{r} \hat{r} + \frac{\hat{r}}{2} (\hat{r} \cdot \nabla \eta) - \frac{1}{2} \nabla \eta, \tag{12}$$

where  $\eta = A^{-1} - 1$ . Applying the Gauss theorem, hence, the energy within radius  $r$  obtained from the Weinberg complex is

$$E_{\text{Weinberg}} = P^0 = \frac{1}{16\pi} \oint Q^{i00} n_i r^2 d\Omega = \frac{r}{2} \eta. \tag{13}$$

The energy component of the covariant energy-momentum four vector of the Weinberg energy-momentum complex is

$$E_{\text{Weinberg}}^{\text{covariant}} = g_{00} E_{\text{Weinberg}} = \frac{\kappa r}{2} = E_{\text{Einstein}}. \tag{14}$$

Subsequently, in the Møller prescription the energy-momentum complex [20] which is given by

$$\Theta_{\nu}^{\mu} = \frac{1}{8\pi} \frac{\partial \chi_{\nu}^{\mu\sigma}}{\partial x^{\sigma}}, \tag{15}$$

where the Møller superpotential

$$\chi_\nu{}^{\mu\sigma} = \sqrt{-g} \left( \frac{\partial g_{\nu\alpha}}{\partial x^\beta} - \frac{\partial g_{\nu\beta}}{\partial x^\alpha} \right) g^{\mu\beta} g^{\sigma\alpha} \tag{16}$$

are quantities antisymmetric in the indices  $\mu, \sigma$ . According to the definition of the Møller energy-momentum complex, the expression for energy is given as

$$E_{\text{Møller}} = \frac{1}{8\pi} \int \frac{\partial \chi_0{}^{0k}}{\partial x^k} d^3x. \tag{17}$$

Notice, that the only nonvanishing component of the Møller energy-momentum complex is

$$\chi_0{}^{0k} = r^2 \sin\theta \frac{dA}{dr}, \tag{18}$$

and the Møller energy within radius  $r$  is

$$E_{\text{Møller}} = \frac{r^2}{2} \frac{dA}{dr} = m \left( 1 + \frac{q^2}{r^2} \right)^{-5/2} \left( 1 - \frac{2q^2}{r^2} \right) - \frac{q^2}{r} \left( 1 + \frac{q^2}{r^2} \right)^{-3} \left( 1 - \frac{q^2}{r^2} \right) \equiv E_2(r). \tag{19}$$

Once more, Ayón-Beato and García show another new regular black hole solution in 1999 [13] with

$$A(r) = 1 - \frac{2mr^2 e^{-q^2/2mr}}{(r^2 + q^2)^{3/2}}, \tag{20}$$

and the parameters  $m$  and  $q$  are associated with mass and charge respectively. For this black hole solution, the Einstein energy complex is

$$E_{\text{Einstein}} = m \left( 1 + \frac{q^2}{r^2} \right)^{-3/2} e^{-q^2/2mr} \equiv E_3(r), \tag{21}$$

and the Møller energy complex obtained by Radinschi [21] is

$$E_{\text{Møller}} = \left[ m \left( 1 + \frac{q^2}{r^2} \right)^{-5/2} \left( 1 - \frac{2q^2}{r^2} \right) - \frac{q^2}{2r} \left( 1 + \frac{q^2}{r^2} \right)^{-3/2} \right] e^{-q^2/2mr} \equiv E_4(r). \tag{22}$$

On the other hand, using the power series expansion, the energy distribution of Einstein energy-momentum complex would become

$$E_1(r) = E_{\text{Tod}} - \frac{3mq^2}{2r^2} + \frac{q^4}{r^3} + \frac{15mq^4}{8r^4} - \frac{3q^6}{2r^5} + \mathcal{O}\left(\frac{1}{r^6}\right), \tag{23}$$

and

$$\begin{aligned} E_3(r) = E_{\text{Tod}} &+ \left( \frac{1}{8m} - \frac{3m}{2q^2} \right) \frac{q^4}{r^2} - \left( \frac{1}{48m^2} - \frac{3}{4q^2} \right) \frac{q^6}{r^3} \\ &+ \left( \frac{1}{384m^3} - \frac{3}{16mq^2} + \frac{15m}{8q^4} \right) \frac{q^8}{r^4} \\ &- \left( \frac{1}{3840m^4} - \frac{1}{32m^2q^2} + \frac{15}{16q^4} \right) \frac{q^{10}}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right), \end{aligned} \tag{24}$$

where the term  $E_{\text{Tod}}$  represents the energy of the Reissner-Nordström solution that corresponds to the Penrose quasi-local mass definition (evaluated by Tod [22]). Also, the energy distribution of Møller energy-momentum complex would become

$$E_2(r) = E_{\text{Komar}} - \frac{9mq^2}{2r^2} + \frac{4q^4}{r^3} + \frac{75mq^4}{8r^4} - \frac{9q^6}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right), \quad (25)$$

and

$$\begin{aligned} E_4(r) = E_{\text{Komar}} + & \left(\frac{3}{8m} - \frac{9m}{2q^2}\right) \frac{q^4}{r^2} - \left(\frac{1}{12m^2} - \frac{3}{q^2}\right) \frac{q^6}{r^3} \\ & + \left(\frac{5}{384m^3} - \frac{15}{16mq^2} + \frac{75m}{8q^4}\right) \frac{q^8}{r^4} \\ & - \left(\frac{1}{640m^4} - \frac{3}{16m^2q^2} + \frac{45}{8q^4}\right) \frac{q^{10}}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right), \end{aligned} \quad (26)$$

where the term  $E_{\text{Komar}}$  agrees with the energy of the Reissner-Nordström solution in the Komar prescription [23]. The first two terms in all expressions are the same as the energy distribution of the Reissner-Nordström solution, and the energy distributions asymptotically behaves as the Reissner-Nordström solution.

### 3 Conclusion

One of the most important themes of general relativity, the energy-momentum localization has not yield a satisfactory solving. At international level, considerable efforts have been made to find a generally accepted expression for the energy-momentum density. Many scientist worked at this issue, using different definitions, like superenergy tensors [24–29], quasi-local mass definitions [30–38], pseudotensorial prescriptions, and here we notice the energy-momentum complexes of Einstein [14–16], Landau-Lifshitz [39], Papapetrou [40–42], Weinberg [17–19] (ELLPW), Bergmann-Thomson [43], Qadir-Sharif [44] and Møller [20], and teleparallel gravity theory [45–55]. The pseudotensorial definitions, except the Møller energy-momentum complex, imply to performed the calculations using non-covariant, coordinate dependent expressions, and yields acceptable results only in the case of quasi-Cartesian coordinates. In the recent years the problem of the usefulness of energy-momentum complexes became a re-opened issue, and many interesting results were obtained [56–89], which demonstrated that these definitions are powerful concepts for energy-momentum localization in general relativity. The (ELLPW), Bergmann-Thompson and Møller prescriptions yield meaningful results in the case of 2 and 3 dimensional space-times [90–93]. We also point out another interesting issue, the similarity of results which were obtained for many geometries using both energy-momentum definitions and the teleparallel gravity theory [45–55]. Whereas Chang, Nester and Chen [30] demonstrated in their studies that the energy-momentum complexes can be considered actually quasi-local and legitimate expressions for the energy-momentum. Their conclusion is that there exist a direct connection between energy-momentum complexes and quasi-local expressions. Furthermore, the new idea of quasi-local approach for energy-momentum complexes [30–34] is the subject of interesting studies and a large class of new pseudotensors connected to the positivity in small regions have been elaborated [35, 36]. The quasi-local quantities can be associated with a closed 2-surface [31–34]. The quasi-local quantities for

finite regions are determined by the Hamiltonian boundary term and the special quasi-local energy-momentum boundary term expressions are connected to physically distinct and geometrically clear boundary conditions [37, 38]. Our paper is focused on the evaluation of the energy distribution for two regular black hole solutions in general relativity coupled to nonlinear electrodynamics given by Ayón-Beato and García [12, 13].

In this paper, according to the definitions of the energy-momentum pseudotensor of Einstein, Weinberg, and Møller, we evaluate the energy distributions of the singularity-free solution, which is obtained by Ayón-Beato and García, from the Einstein equations coupling to a nonlinear electrodynamics. Also, the Einstein and Weinberg energy-momentum complex have the same results, and the Møller energy-momentum complex gives a different result. In the limit of a vanishing charge  $q$ , we obtain in the Einstein, Weinberg and Møller prescription the same result, the energy is the mass of the black hole, and this is the same as the expression for the energy of the Schwarzschild solution. Here, Vagenas [94] hypothesizes that there is a relation

$$\alpha_n^{(\text{Einstein})} = \frac{1}{n + 1} \alpha_n^{(\text{Møller})}, \tag{27}$$

between the coefficients of the expression for energy in the Einstein prescription

$$E_{\text{Einstein}} = \sum_{n=0}^{\infty} \alpha_n^{(\text{Einstein})} r^{-n} \tag{28}$$

and in the Møller prescription

$$E_{\text{Møller}} = \sum_{n=0}^{\infty} \alpha_n^{(\text{Møller})} r^{-n}. \tag{29}$$

To compare with (23), (24), (25) and (26), our results support Vagenas hypothesize. Although, the relation can be understood the product of the formula [95]

$$E_{\text{Møller}} = E_{\text{Einstein}} - r \frac{dE_{\text{Einstein}}}{dr}, \tag{30}$$

which will be derived from (8) and (19). Furthermore, we make a comparison with the expression for energy of Einstein and Møller energy-momentum complexes obtained by Radinschi [96, 97]

$$\bar{E}_{\text{Einstein}} = \bar{E}_{\text{Weinberg}} = E_{\text{Tod}} + \frac{q^6}{24m^2r^3} - \frac{q^{10}}{240m^4r^5} + \mathcal{O}\left(\frac{1}{r^6}\right) \tag{31}$$

and

$$\bar{E}_{\text{Møller}} = E_{\text{Komar}} + \frac{q^6}{6m^2r^3} - \frac{q^{10}}{40m^4r^5} + \mathcal{O}\left(\frac{1}{r^6}\right). \tag{32}$$

These results are evaluated for another regular black hole solution which is also suggested by Ayón-Beato and García [98]. Notice to the difference in the third term between these two solutions, it could be used to survey the factualness of solutions.

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